MATH TO WORK

Sol Garfunkel
COMAP

There are a few things that I should mention explicitly, so that it is clear as to the attitudes and biases that shape my thoughts. I am a mathematician by training and I believe in the beauty as well as the applicability of the subject. I do NOT believe that mathematics should be taught solely from the point of view of its applications, either in general or more specifically as applied to work. However, I strongly believe that mathematics should be taught in context wherever possible – for a number of reasons. Foremost among these is that in many cases that is why the mathematics was developed – to analyze and solve real problems. Without gambling it is unlikely that Descartes would have worked on probability when he did. Without WWII and the need to efficiently move men and materiel it is unlikely that Von Neumann would have worked on operations research or for that matter Turing work on codes. To teach these subjects out of context would be dry swimming indeed.

Throughout this paper I refer to mathematical modeling. To me this is a life skill, different in kind from the mastery of any particular algorithm (no matter how useful). Modeling is the process by which we first determine whether a problem we are presented with can be attacked using mathematics. Real problems almost never involve only one branch of mathematics. They are not applications as we typically teach them, i.e. a problem that comes at the end of a chapter in a mathematics text, so that students know they are to use the method just learned. In the real world problems don't tell you that they are algebraic or geometric or statistical in nature. And most of the time, they are a combination of these.

Modeling is a technique that can (and in my opinion should) be introduced and reinforced from elementary school on. This is doable. The idea, for example, that a graph can represent a network of city streets can clearly be introduced early, as we show students metro maps as representations (models) of actual underground transportation systems. I don't see this as teaching mathematics for work, but giving children early exposure to different forms of representation. Taxicab distance (measuring along streets) is as interesting mathematically as Euclidean distance and extremely useful when you want to decide the best location for a fire station (assuming you don't allow fire trucks to frequently ride across people's lawns).

I was not asked to address questions of pedagogy and assessment in this paper, but rather to focus on content. But for what it's worth I believe that mathematics should be taught to students in a manner that reflects how they might be called upon to use it. People generally work together on problems, hence all the experimentation with collaborative learning. People are almost always required to communicate their work to others – hence the need to gain an ability to read and write mathematically and make convincing mathematical arguments (often geared to the person you are trying to convince). And since I believe that we must assess what we value, I believe that assessment must be

broad enough to test these abilities of communication as well. Whether or not I mention these explicitly again in the paper, these are the beliefs that underlie my thesis.

When we design curricula and syllabi for mathematics offerings in both high schools and colleges all too often we do not take the needs of the students into account. There is a loss of the forest for the trees phenomena at work here which we fail to recognize at our peril. We begin with a set of assumptions (frequently outdated) and back up from there. An example: Calculus is important for students to succeed (in school, work, life). Algebraic symbol manipulation is important for success in calculus. Therefore, algebra is important for success in school, life, work. In order to be successful with algebraic symbol manipulation students must have facility with arithmetic operations, particularly the addition of fractions. Therefore, the addition of fractions is necessary for a successful life. Thus we gear our elementary school mathematics curriculum for mastery of arithmetic operations, push work on algebra down to middle schools, test algebraic symbol manipulation to death in high school, and hope that every student will take and pass an AP calculus exam.

What's wrong with this picture? Several things, of course, but most notably the fact that the underlying assumption is simply no longer true and hasn't been true for some time. As a mathematician, I recognize the beauty and centrality of calculus, but it should be more than clear by now that in terms of applications of mathematics in the work force, in daily life, for good citizenship and even for success in further academic studies other branches of mathematics along with the processes of mathematical modeling are increasingly more relevant. Again this must be about the needs of students. We know that their future will involve many different jobs and the need to master current and emerging technologies. We know that they will need creativity, independence, imagination, and problem-solving abilities in addition to skills proficiency. In other words, students will increasingly need mathematical understanding and awareness of the tools mathematics provides in order to achieve their career goals.

Compare this to the present, rather cynical, state of affairs, especially for advanced students. Half the number of students who take an AP calculus course take the AP exam. Half the students who take the AP calculus exam receive a passing grade. Half the students who receive a passing grade in AP calculus never take another math class again. Think about this. In the name of preparing students for their quantitative needs at school, life, and work, we spend 12 years gearing their mathematics education to succeeding in calculus (or more accurately on one specific calculus test) and the students use that test simply as a way to impress college entrance committees and pad their applications. Four years later when they prepare to join the work place or go on to graduate studies, how much of that mathematics will they be able to draw on?

Continuing this shell game serves no one. Students have a real need for mathematics. Being able to analyze and solve problems using quantitative reasoning is an increasingly necessary job skill. And while we focus here mainly on the work force, it is also abundantly clear that mathematical models are at the heart of much of the decision-making we are called upon as citizens to make – from climate change to the ethics of

genomics to deciding elections. Public understanding of these models is a prerequisite for maintaining a true Jeffersonian democracy.

Where are we now? For non-STEM majors, their last undergraduate mathematics course is almost universally a College Algebra offering. It should be noted that the 'College' in the course title was added so as not to embarrass students who could clearly see that this was in fact high school material. This course was originally designed to help prepare students who did not do sufficiently well in high school to go on in college to a standard calculus sequence. It doesn't. It is their last course, much in the way that Algebra II is most often the last high school course; yet the content of these courses only makes sense for students who will be continuing their mathematical studies – certainly not for students who will go from there to the world of work. We know this and have nonetheless done little to change the status quo, preferring to operate from the unproven notion that studying any mathematics is somehow a beneficial exercise for the mind.

There is, of course, a case for calculus (and by implication, the algebraic skills necessary to succeed in calculus). My argument here is that the design of the high school curriculum and to a large extent the college mathematics curriculum assumes a path to and through calculus and that assumption is no longer valid for the majority of students. In general, the mathematics curriculum suffers from the fact that every course has as its main justification the fact that it is prerequisite for the next course, while in point of fact, the half-life of students in mathematics course is still pretty close to one year, i.e. each year past 10^{th} grade we lose approximately half the student enrollment. Having said that, I am not arguing that there is no place for calculus and hence no place for algebra – only that we need a better balance between the ideas we present. We are still the only industrialized country with the layer-cake curricula of algebra, geometry, algebra rather than a more integrated approach. And yes, there is a subtlety here – namely that at some point curricula may diverge into continuous (calculus, differential equations,…) and discrete (operations research, statistics,…) tracks. But as anyone who has gone through proofs of Arrow's impossibility theorem for voting methods can tell you, these are not necessarily divisions by level of difficulty.

For the moment, let us leave aside questions of implementation, i.e. how to move away from entrenched curricula and textbooks and look first at students' needs, in particular what they will need when they enter the world of work. On the surface this looks as though it would be very different for students entering the work place immediately after high school as opposed to those entering with an associates or bachelors degree. But for many positions those differences may not be very great. And in the present mode whether a student leaves high school having last taken Algebra II or college having last taken College Algebra, there is little to no difference in content knowledge.

I need to begin this discussion with a caveat. One size does not fit all. There is work to be done, industry-by-industry, to look hard at the quantitative reasoning and skills needed for employees now and in the foreseeable future. In an ideal world we could build courses or sequences of courses designed to effectively prepare students for that work with sufficient examples and case studies taken directly from work experiences. What I

will address here are ideas and topics that I believe could reasonably be put in any such courses, regardless of vocation. These conclusions are based in part on my experience as project director of a three-year project, funded by the National Science Foundation (NSF), entitled WorkMap.

WorkMap was funded under NSF's Advanced Technological Education (ATE) section that is mainly concerned with technology education, especially, but not exclusively, as it occurs at two-year colleges. In almost all associates degree programs, including those in biotechnology, allied health, environmental technology, and manufacturing technology, students' last math course is College Algebra. This course is invariably taught in the college mathematics department and is geared for transfer credit to four - year institutions, despite the fact that many if not most of the students will go into the work force after two years. In many ways this group of students epitomizes the mismatch between what we teach and who we teach.

In the WorkMap project a team of mathematicians and educational researchers visited some fifteen companies (including a hospital, an international speedway, a financial services company, an airplane manufacturer, a major software developer, and an automobile association), shadowing and interviewing entry-level workers, their immediate superiors, and human resource personnel to see what mathematical reasoning and skills were employed on the job. Not surprisingly, we did not see algebra at the operational level. Rather we saw the ideas from operations research – scheduling, inventory, queuing, as well as the statistical reasoning used in quality control, and three-dimensional visualization necessary to use and understand new software. Yet these ideas are barely in the curriculum for STEM and non-STEM majors. It is well past time for this to change.

In discussion with HR people we learned that while they certainly rely on school and college transcripts, many administer their own tests. And as the January 2008 report from Peter D. Hart Research Associates, entitled, 'How Should Colleges Assess and Improve Student Learning' points out, "When it comes to the assessment practices that employers trust to indicate a graduate's level of knowledge and potential to succeed in the job world, employers dismiss tests of general content knowledge in favor of assessments of real-world and applied learning approaches."

When we began the WorkMap study we agreed to focus on IT workplaces. During the course of the study it became clear just how broad a category that was. Almost every large entity is an IT workplace today. In many ways this speaks to the importance of the work. We also agreed to concentrate on entry-level employees and specifically in jobs where an associate's degree was an appropriate job qualification, in order to center our work in the ATE world. Of course, this was somewhat idealized. In some cities and industries we found a buyer's market, where employers had no difficulty hiring, even for entry-level jobs, people with several years of experience and higher-level degrees.

The jobs we observed for the main part fell into the following categories: technical support services, help desk, quality control. With the exception of some quality control

personnel who directly used statistical techniques and software our observations of the uses of mathematics were of the use of mathematical reasoning. Again, the mathematicians on the team expected this state of affairs and were most interested in looking harder at the kinds of reasoning involved. What we observed was operations research at work. We saw instances of scheduling, inventory, queuing theory, bin packing. We do not mean to imply that the employees were engaged in using Kruskal's algorithm* or constructing minimum-cost spanning trees – only that these ideas were behind the reasoning involved.

For instance in the technical support and help desk world, someone reports that a computer doesn't function properly or that a piece of software is giving anomalous results. The support person has to first make a decision on whether the problem is one that he/she has the ability to solve. If the answer is a tentative yes, then there are a series of steps they suggest the client go through in order to solve the problem – all the while aware (in most cases) that if a problem takes too much time there is another price to pay. With respect to both tech support and help desk activity, there is almost always a queue. And while some situations may be handled on a first come first serve basis, in others there is a hierarchy of clients or other reasons why some problems need to be handled before others. As an aside (but an important one), it was a pleasant surprise to see the level of intellectual honesty of the employees observed. They very much needed to know what they knew and what they didn't. The decision to carry on with a problem or pass it on to someone more knowledgeable was crucial to their efficiency and success.
Another somewhat meta-observation is that when specific mathematical techniques were employed, the successful end result was one that satisfied the immediate supervisor, rather than the most 'elegant' from a mathematical point of view. We saw this most clearly early in the project with the creation of an effective back up plan for storage discs. The plan that was implemented was far from the most efficient from the mathematical point of view; but there were tables and charts aplenty which convinced the immediate supervisor that this was a solution, which would guarantee sufficient backup.
We should also mention 3-D visualization. A number of software packages either being used or designed by the employees we observed required a facility with 3-D visualization. Again, no specific techniques or theorems from solid geometry were needed, but a comfort level with manipulating parameters and seeing how the screen setting was affected was a natural part of several people's work.

With respect to curricula, the mathematics entailed here falls into the realm of discrete mathematics. In particular, it does not require calculus in order to achieve significant understandings. Certainly there would need to be a heavier reliance on both probability and statistics, but this is a trend that has been on the rise for the past several years. Students are beginning to see statistical ideas in middle school, if not earlier. What is attractive here is that much of the elementary work in operations research, while far from trivial, does not rely heavily on algebraic symbol manipulation. This needs to be said again, so there is no mistake. Discrete mathematics, statistical argument, and three-dimensional visualization are not easy topics. They are not in any sense easier than algebra. They are simply different. It is certainly not a sure thing that students who fare poorly in algebra related courses will necessarily do better in OR related courses. It is

simply that for a large number of students these ideas, which are as mathematically and academically important as those related to algebra and calculus, may be more useful. And the word useful must be understood in context. A very small segment of the work force will be called upon to DO mathematics, but a very large percentage may be called on to reason mathematically. And much of that reasoning will be discrete in nature.

To be clear, there is a case for an algebra to calculus approach for some students. Many mathematical models, especially of physical phenomena depend upon an understanding of differential equations. For engineers and epidemiologists among others continuous mathematics is essential. For them the early work on algebra is clearly helpful and necessary. But for the large majority of students it is not. Algebra in high school and calculus in college become artificial barriers to advancement. There is a false argument that has achieved (pun intended) widespread acceptance that must be recognized and rejected. The argument goes like this. All students need to learn mathematics (so far so good). We shouldn't discriminate against any group of students (still hard to argue). All students must be given the opportunity to reach some basic level of mathematical competence. That basic level of mathematical competence can be defined by the content of Algebra II (as exhibited on a particular test). Criminal!

In the name of giving everyone an equal chance to succeed, we merely give them an equal chance to fail. It is a debatable point as to whether (past simple arithmetic) there is a fixed body of mathematical knowledge that every student should know. But in any case it shouldn't be a series of manipulative exercises that students will forget as soon as their last test on them is taken and which do not relate to the work skills they will need or to their lives as productive citizens. The simple truth is that there is an enormous choice of mathematical topics we could (and should) be teaching. And these will change as well.

I have participated in what seems like hundreds of conversations about the history of mathematics education and I am struck by the fact that these conversations never discuss the future of mathematics or the future of the applications of mathematics. The subject is growing. The areas in which mathematics is being effectively applied are growing. The technologies we can draw on are exploding. In this environment, what sense does it make to have discussions of the future of math education devolve into how can we better teach students to add fractions. We should be asking how can our curricula and syllabi change to meet the changing needs of our subject and most importantly – the changing needs of our students.

So, we arguing here for curricular alternatives – high school and college courses emphasizing discrete ideas taken from statistics, geometry, and operations research with case studies and applications to a variety of disciplines, work place settings as well as the kind of social decision making all of us will face. In many ways this is the easy part. Good material abounds and there are curriculum developers (this author included) ready and willing to produce the texts, software, etc needed to make this a reality. Now we have to think about implementation and some of the barriers to change.

As in all discussions of curricula change, we have to find effective means for staff development. Here, I feel quite optimistic. My optimism is borne from our experiences with the Math for Liberal Arts text, 'For All Practical Purposes'. This text was first published in 1987 and is about to go into its 8th edition. It has sold well over 250K copies and has been the leading selling text in its market from its first day of publication. I point this out because the table of contents of the text has zero intersection with the tables of contents of the traditional Math for Liberal Arts texts, including a fair amount of discrete topics. Examples include: Election Theory, Game Theory, Machine Scheduling, Bin Packing, Fair Division, etc. Given the fact that most instructors had never seen any of this content until they taught from the book, its immediate success was a gratifying surprise. Moreover, the course is typically taught in many sections by either graduate students at universities or adjunct faculty at colleges. Ball State for example offers 55 sections of the course in the fall. They run summer training sessions for their adjunct faculty and monitor their work throughout the year. I have given a number of talks at Ball State and other colleges that have adopted the book. I have asked the faculty why they were willing to make the investment of time and effort to master so much new material. Their answers were basically the same. They felt that what they were teaching didn't work and that the new material was sufficiently interesting and important to be worth the effort.

I firmly believe that if we are to make the changes suggested here – offer serious alternatives to the algebra-calculus sequence, then that change must begin at the college level. The good news is that from a staff development point of view this should be doable. Since For All Practical Purposes and its clones and the increased emphasis on the teaching of statistics, college faculty (including adjuncts) should be able to get quickly up to speed. But the acceptance of these ideas, i.e. the politics is another matter. There are nonetheless encouraging signs.

The most recent Committee on the Undergraduate Program (CUPM) report and the work of the MAA's Foundation Project point to a general acceptance that College Algebra must change. My sense is that the leadership of MAA and AMATYC are on the same page. But more push back will be needed to make that change actually happen. It is my feeling that that push back must come from the world of work. Academia simply moves slowly and math departments slower than that. On the other hand, there are some 1200 math departments in the country with 6 or fewer full-time faculty. Many of these departments are in colleges in towns with one or two major employers. They must per force be responsive to the needs of local industry. This suggests a bottom up approach as well as the top down provided by the professional societies.

I mentioned that the colleges must act first, if we are to have change at the school level. Colleges can give lip service day and night to any set of ideas, but admissions criteria and college placement tests rule! Students and parents are less interested in what colleges say than what they do. Any alternative path into mathematics must be valued and endorsed at the college level if we are to expect any change at the secondary level. And parents must be brought along so that the change is seen to make sense and to benefit their children.

If we imagine a world in which these new college courses have been designed and implemented, in which the colleges make clear that these courses and the prerequisite school work will be valued (by placing appropriate questions on placement tests), then how do we make the requisite change in school mathematics? Again, getting material written is not a serious problem. Staff development will be. One can work hard at pre-service training to insure that would be math teachers take the new material at the college level so that they are prepared to teach it at the high school level. But there is no doubt that significant in-service work will be necessary.

Again there will be political barriers to acceptance at the school level. Much will depend upon the willingness of colleges and businesses to make the case locally and the professional societies to make the case nationally. This leads me to some recommendations to the Commission. As I have said, I believe that there should be both a top down and bottom up approach or perhaps more accurately, a national and local approach. At the national level I envision a conference of academic, industry, and government leaders along with representatives of various funding agencies to set up some overriding goals for the work. But at the local level, I think that we need a series of experiments/case studies. In particular, I envision towns with a few major employers working with, for example, one local two-year and one local four-year college and feeder high schools to create and implement new course sequences. This could also be done by field – allied health or nursing or biotech to create models which can be more easily scaled up.

Depending upon the availability of funds, specific curricula development projects could be funded or co-funded with agencies such as NSF. There is some precedent here. In the mid-80's the Sloan Foundation funded a conference to develop alternative curricula and teaching methods for calculus. A report of that conference, "Toward a Lean and Lively Calculus" appeared in the MAA notes series. This was followed by a convocation, "Calculus for a New Century" at the National Academy of Sciences. Based on this work, NSF funded several major calculus reform projects from 1988-1994 for somewhat over $25 million. Perhaps a similar model could work here, especially given the upcoming change in administration.

Under any circumstance, it is well past time for industry to speak out and be heard. I welcome the opportunity to share these thoughts and look forward to working with you in the future.

* Kruskal's algorithm is an algorithm in graph theory that finds, for a connected weighted graph, a subset of the edges of the graph that forms a tree that includes every vertex, where the total weight of all the edges is minimized.